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# The effect of layer depth on mixing in a double-diffusive two-layer system

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**Abstract**

Experiments were carried out to investigate the effect of the layer depth ratio on the mixing process of a two-layer stratified fluid in a laterally heated enclosure. Due to the lateral heating of the enclosure, a circulating convective flow is induced in each layer. The experiments were carried out in a box with inner dimensions of $10 \times 10 \times 10$ cm$^3$. Two sidewalls of the box were made of stainless steel and served as heat exchangers, and the two other sidewalls were made of optical glass to facilitate flow visualization. The results show that when one layer is deeper than the other, the mixing time is reduced compared to its value in the case of equal-depth layers. When the top layer is deeper, the mixing times are generally larger than those for a deeper bottom layer. This indicates on an asymmetry in the system. © 2002 Elsevier Science Ltd. All rights reserved.

*Keywords:* Double diffusion; Natural convection; Stratified

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## 1. Introduction

One of the fundamental double-diffusive systems is a two-layer solute-stratified fluid in a laterally heated enclosure. Several research works were focused on the stability of the interfacial flow and the mixing process of the two layers. Bergman and Ungan [1] studied experimentally the behavior of a two-layer system and measured the mixing time associated with layer merging. They have shown that for a given buoyancy ratio (defined as the ratio between the stabilizing density step due to salinity and the lateral destabilizing density difference due to temperature), the nondimensional mixing time decreases as the lateral thermal Rayleigh number is increased. For a given thermal Rayleigh number, the mixing time increases with the buoyancy ratio. Hyun and Bergman [2] conducted a numerical simulation of the above phenomenon. The mixing times estimated on the basis of the simulation were in approximate agreement with those measured in their experiments.

Recently, Tanny and Yakubov [3] reported on an experimental study in which the stability of the flow adjacent to the interface was examined and the associated mixing time of the system was measured. Their study revealed that the interfacial flow can be stable or unstable and the stability criteria were determined in terms of the governing parameters of the problem. Experiments under unstable conditions were characterized by the existence of vortices, moving along the interface above and below it. On the other hand, in experiments under stable conditions no vortices were observed. Fig. 1 is a Schlieren picture illustrating the vortices formed in an experiment under unstable conditions. This picture is reproduced from [3]. Tanny and Yakubov [3] have also shown that under stable conditions the mixing time of the two-layer system is much larger than under unstable ones.

All the above studies considered two layers of equal depth, i.e., the layer depth ratio, $L_r = H_t/H_b = 1$. Here $H_t$ and $H_b$ are the depths of the top and bottom layers, respectively. Linden [4] studied the formation and merging of a series of double diffusive layers in a stable

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**Nomenclature**

| | |
|---|---|
| $\Delta C$ | initial concentration difference between the two layers |
| $D$ | coefficient of solute diffusivity |
| $g$ | acceleration due to gravity |
| $H_b$ | depth of bottom layer |
| $H_t$ | depth of top layer |
| $L$ | width of the enclosure |
| $L_r = H_t/H_b$ | ratio of layer depth |
| $Ra = g\alpha\Delta TL^3/\nu\kappa$ | Rayleigh number |
| $R_\rho = \beta\Delta C/\alpha\Delta T$ | buoyancy ratio |
| $t$ | dimensional time |
| $\Delta T$ | lateral temperature difference |
| *Greek symbols* | |
| $\alpha$ | coefficient of thermal expansion |
| $\beta$ | coefficient of solutal contraction |
| $\kappa$ | coefficient of heat diffusivity |
| $\nu$ | kinematic viscosity |
| *Subscripts* | |
| m | mixing |

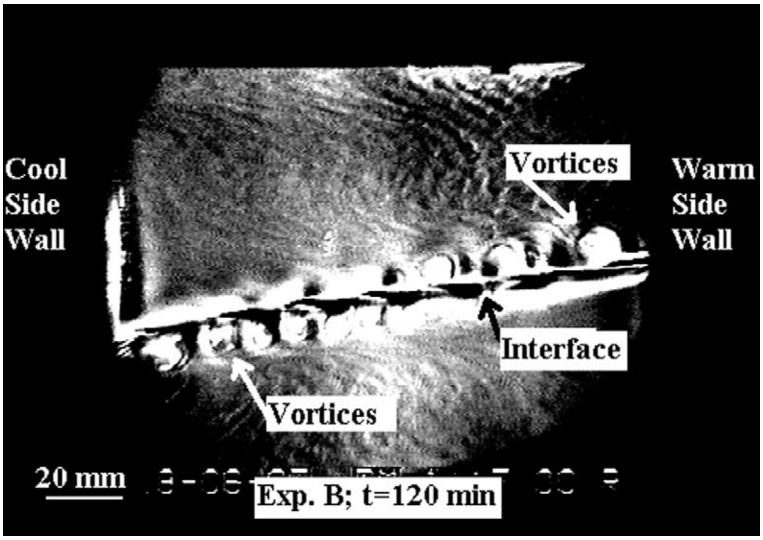

Fig. 1. Schlieren flow visualization of the vortices moving above and below the interface in an experiment under unstable conditions. $\Delta C = 0.2\%$, $\Delta T = 10$ °C. Reproduced from [3].

solute gradient heated (cooled) from below (above). He noted that when the thickness of two adjacent layers is not equal, the merging process of these layers is faster. Linden explained that due to the different thickness of the layers, an imbalance exists in the convective velocity on both sides of the interface. This imbalance causes faster transport through the interface and thus a shorter mixing process. He also observed that during the merging process, the shallower of the two layers becomes even shallower.

The aim of the present work is to explore the effect of layer depth ratio on the mixing of a two-layer solute stratified fluid in a laterally heated enclosure. Two different configurations are examined, depending on the relative thickness of the top and bottom layers. The first is Shallow-Over-Deep (SOD) layers, where the top layer is shallower than the bottom one and the second is Deep-Over-Shallow (DOS) layers. Under ideal conditions, the two cases are symmetric. However, our experimental results described below suggest that in practice they are not.

## 2. Experimental set-up and procedures

The experimental apparatus and procedures are essentially similar to those used in [3]. For convenience, they are described shortly below. The experiments were carried out in a box with inner dimensions of $10 \times 10 \times 10$ cm$^3$. Two sidewalls of the box were made of stainless steel and served as heat exchangers, and the two other sidewalls were made of optical glass to facilitate Schlieren flow visualization. The metal side walls and the top and base of the box were insulated using Styrofoam plates. The optical glass sidewalls were insulated by transparent thermal insulation units. The temperature of each sidewall was measured by 5 miniature thermocouples type T, inserted into holes from the backside of the metal wall.

The flow was visualized using a Schlieren system consisting of two spherical mirrors, 15.24 cm in diameter and 152.4 cm in focal length. The output of the Schlieren system was imaged by a CCD camera, and was displayed on a monitor and also recorded by a time-lapse VCR for later reviewing.

The two-layer stratified system was established by initially filling the tank with a lower layer of aqueous salt (NaCl) solution of the higher concentration. Then, an upper layer of fresh water was poured carefully (using a wooden float). The lateral prescribed temperature difference was applied by raising and lowering the temperatures of the sidewalls, and was kept constant throughout the experiment. The temperature of each side wall was found to be uniform within ±0.5 °C. Each experiment continued until the two-layer stratified fluid was mixed through the "catastrophe" stage [3].

The thickness of each layer was varied between 1 and 8 cm while the total fluid depth was kept constant at 9 cm. Hence, for the SOD case, experiments were done with $L_r = H_t/H_b = 0.125$, 0.2, 0.286, 0.385, 0.5, 0.636, 0.8 and 1. For the DOS case experiments where carried out with $L_r = 1.25$, 1.57, 2, 2.6, 3.5 and 8.

All the experiments were conducted at the same concentration and temperature differences. The concentrations of the bottom and top layers were 0.5 and 0 wt%, respectively. The temperatures of the warm and cool sidewalls were 35 and 10 °C, respectively. These values can be represented in terms of the nondimensional governing parameters of the problem. The thermal Rayleigh number is defined as

$$Ra = \frac{g\alpha\Delta T L^3}{\nu\kappa}, \tag{1}$$

where $g$ is the acceleration due to gravity, $\alpha$ is the coefficient of thermal expansion, $\Delta T$ is the lateral temperature difference, $L$ is the box width (distance between warm and cool side walls), $\nu$ is the kinematic viscosity and $\kappa$ is the coefficient of thermal diffusion. The ratio between the stabilizing effect of solute concentration and the destabilizing effect of the temperature gradient is represented by the overall buoyancy ratio:

$$R_\rho = \frac{\beta\Delta C}{\alpha\Delta T}, \tag{2}$$

where $\beta$ is the coefficient of solutal expansion and $\Delta C$ is the initial concentration difference between the two layers. The physical properties of the fluid were estimated at the mean temperature and concentration in the system, using the data given in [5–7].

A total of 20 experiments was carried out, 13 with $L_r \leqslant 1$ (SOD) and 7 with $L_r > 1$ (DOS). For these 20 experiments, the average value of the thermal Rayleigh number is $(4.04 \pm 0.13) \times 10^8$ and the average value of the buoyancy ratio $R_\rho$ is $0.64 \pm 0.027$. These values correspond to unstable condition [3] in which vortices exist above and below the interface during the whole experiment.

## 3. Results and discussion

In Fig. 2 we present the interface position as a function of the time elapsed from the start of the lateral heating for several experiments with different $L_r$. As can be seen in Fig. 2 the mixing process consists of two stages. The initial, quasi-steady stage, is a relatively long period, during which the interface is almost stationary. During this period, vortices exist above and below the interface (e.g. Fig. 1) and transfer heat and salt across it. This stage terminates as the interface breaks down near the warm sidewall and fluid starts to penetrate from one layer to the other. This penetration starts the second stage, which is a shorter period of interface migration towards the top or bottom of the tank (the 'catastrophe' stage [3]). At the end of the second stage the interface disappears and, due to the flow of fluid from one layer to the other, salt fingers fill the whole fluid.

Our observations of interface migration (Fig. 2) show that for all the experiments with $L_r \geqslant 0.5$ (both SOD and DOS), the interface migrated downwards, while for $L_r < 0.5$ (SOD only), it migrated upwards. Hence, in the DOS configuration the shallower layer (the bottom one) always becomes even shallower. However, in the SOD configuration this rule is valid only when the upper layer is

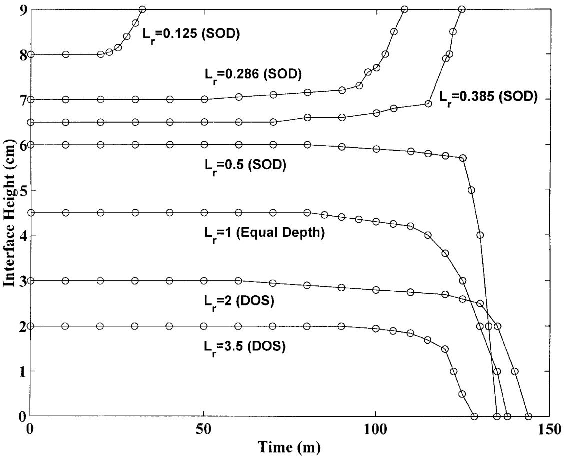

Fig. 2. The position of the density interface as a function of the time elapsed from the start of differential heating. The curves illustrate the initial quasi-steady stage with an almost stationary interface and the second stage of interface migration towards the top or bottom of the enclosure. The different curves represent different values of the initial $L_r$.

thin enough, namely, when $L_r < 0.5$. For thicker upper layers, $0.5 \leqslant L_r < 1$, the situation is reversed and the initially deeper (bottom) layer becomes shallower during the downwards interface migration. It should be emphasized that the criterion for the change in the migration direction, $L_r = 0.5$, is approximate since not enough experiments were carried out to identify this limit precisely.

Kamakura and Ozoe [8] suggested that, due to the effect of temperature on fluid properties, the interface in a system with two equal-depth layers would always migrate upwards. They also showed that the interface migrates continuously throughout the experiment (see their Fig. 6), in contrast to the two-stage process observed in our study. Their findings are different from our results on interface migration as mentioned above and displayed in Fig. 2. The differences between the results may be due to the following reasons: (i) They considered a narrow tall enclosure, with aspect ratio (height/width) of 4, while our aspect ratio is 0.9. (ii) Their operating conditions ($Ra$ and $R_\rho$) were such that the interfacial flow was stable, unlike the unstable flow in our present experiments with vortices moving along the interface above and below it. This suggests that the migration process in a two-layer system strongly depend on geometry and physical conditions.

Linden [4] observed that interface migration was always such that the shallower layer became even shallower, regardless of whether the shallower layer is initially above or below the deeper one. Linden arrived at this conclusion on the basis of two observations. The first is in the DOS configuration (see his Figs. 6(e)–(g)) and the second is in the SOD configuration (see his Figs. 6(h)–(j)) for $L_r \approx 0.5$. Hence, his observations are in agreement with ours.

In this work, as in [3], we define the mixing time as the time elapsed from the start of the lateral heating until the whole fluid was filled with salt fingers. The mixing time, $t_m$, was normalized by a characteristic time scale, $L^2/D$, where $D$ is the coefficient of solute diffusivity and $L$ is the enclosure width. The time scale is chosen with the coefficient of solute diffusivity (and not the heat diffusivity) since mixing of the two layers is associated with the approach of $\Delta C$ to zero [1].

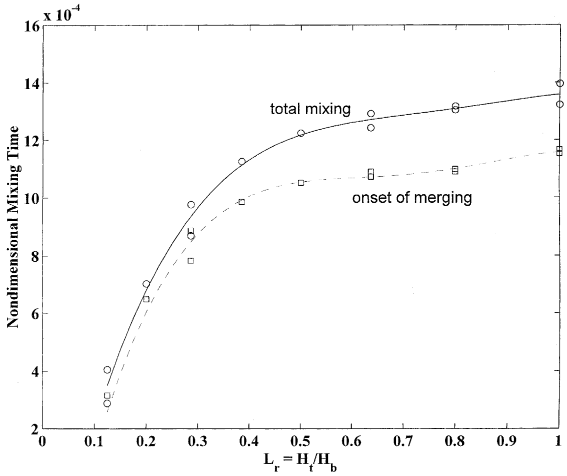

Fig. 3. The nondimensional mixing time as a function of the layer depth ratio, $L_r$, for the case of Shallow-Over-Deep (SOD) layers. The circles (○) represent the total mixing time while the squares (□) represent the time elapsed until the onset of the merging process. The solid and dashed lines were fitted to the data points by the least-squares method.

The nondimensional mixing time as a function of $L_r$ is shown in Fig. 3 for the 13 experiments with SOD layers, $L_r \leqslant 1$. The solid line in Fig. 3 represents the total time elapsed until salt fingers fill the whole tank. It is clearly observed that this time increases monotonously as $L_r$ approaches 1, i.e., as the layers approach equal initial thickness. The dashed line in Fig. 3 represents an earlier event, namely, the onset of the merging process (for more details of the process see [3]). This event is usually characterized by the disappearance of the vortices on one side of the interface and the penetration of fluid from one layer to the other (i.e. interface breakdown). Until that moment, vortices exist on both sides of the interface and the interface is almost stationary (see Fig. 2). Hence, the time difference between the dashed and solid lines represents the process of interface migration. It is observed that this time difference increases with $L_r$, probably due to the longer way the interface has to migrate when its initial location is closer to the mid-height of the tank ($L_r = 1$).

This finding is in general agreement with Linden's [4] observations. He noticed that when one layer is shallower than the other, the interface migration is faster, due to the imbalance in the convective velocities on both sides of the interface. Our results suggest that the imbalance between the velocities in the two layers affects the whole mixing process, including the quasi-steady stage of a stationary interface, and not only the final shorter stage of interface migration.

One would expect that due to symmetry, the mixing time in the case of DOS layers would change with $L_r$ in exactly the same manner as in the opposite case of SOD layers. Our study suggests that in practice this is not the case. Fig. 4 shows a comparison between the mixing times measured in the two cases. For that comparison only the time of complete mixing is shown. For convenience, the time for the DOS case is presented as a function of $1/L_r$, so that the same horizontal scale applies for both cases, $1/L_r|_{(\mathrm{DOS})} = L_r|_{(\mathrm{SOD})}$.

In Fig. 4 the circles represent the mixing time for the SOD case (as in Fig. 3) while the '×' symbols represent the DOS case. Qualitatively, the results show that in both cases the mixing time increases as the layers approach equal depth. However, the mixing time for the DOS case is generally larger than that for the SOD case,

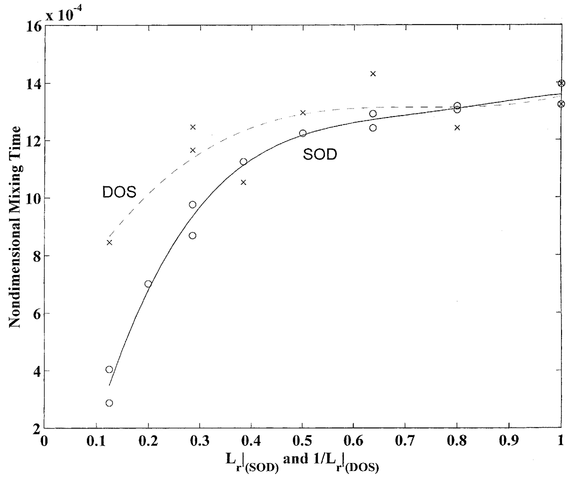

Fig. 4. A comparison between the nondimensional mixing times in the Shallow-Over-Deep (SOD) and the Deep-Over-Shallow (DOS) cases. (○) SOD; (×) DOS. The solid and dashed lines were fitted to the data points by the least-squares method.

as illustrated by the dashed and solid lines, respectively in Fig. 4. In this sense the system is asymmetric. The difference in the mixing times increases with the asymmetry in the layer depth, and it is most pronounced at $1/L_r|_{(DOS)} = 0.125$ where the mixing time for the DOS case is more than twice that for the SOD case.

Another striking difference between the two cases is that in contrast to the monotonous increase of the mixing time in the SOD case, the behavior in the DOS case in not monotonous, as shown by the '×' symbols in Fig. 4. The nonmonotonous mixing times obtained in the DOS case imply on a chaotic behavior of the two-layer system in this configuration. A chaotic merging process of double-diffusive layers was already observed [9] when a stable solute gradient was heated at a single vertical sidewall.

The asymmetry between the mixing times for the SOD and DOS cases (Fig. 4) can be attributed to the effect of the temperature-dependent fluid properties on the system's behavior. For the aqueous solution under study, the kinematic viscosity ($\nu$) decreases with temperature and the coefficient of thermal expansion ($\alpha$) increases with temperature. Hence, the local Rayleigh number increases with temperature. Therefore the local upwards fluid velocity near the warm sidewall and just below the interface (for a warm wall on the right) would be higher than the downwards velocity near the cool sidewall and just above the interface.

Flow visualization suggests [3] that the vertical flow along the sidewall, which impinges the interface, plays a major role in the mixing process. This flow causes the formation of the vortices (Fig. 1) that move along the interface above and below it. The vortices transfer heat and salt across the interface and hence, more intense vortices will cause a faster mixing process. When the bottom layer is deeper (SOD), the upwards velocity along the warm sidewall is relatively high and the mixing process is fast. In the opposite case, when the top layer is deeper (DOS), the downwards velocity along the cool sidewall is not as fast, due to the lower local Rayleigh number near the cool sidewall. Consequently, under otherwise similar conditions, the mixing time in the DOS configuration would be higher than that in the SOD configuration.

## 4. Conclusions

Experiments were conducted to investigate the effect of layer-depth ratio on the mixing process of a two-layer stratified fluid in a laterally heated enclosure with aspect ratio of 0.9. The following conclusions can be drawn from the results of the experiments:

- The mixing time of the system increases as the layers approach equal depth.
- The mixing time is larger for the case of DOS layers than for the opposite case of SOD layers. In this sense the system is asymmetric.

## Acknowledgements

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